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HERD BEHAVIOUR, BUBBLES AND CRASHES*

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This paper attempts to formalise herd behaviour or mutual mimetic contagion in speculative markets. The emergence of bubbles is explained as a self-organising process of infection among traders leading to equilibrium prices which deviate from fundamental values. It is postulated furthermore that the speculators' readiness to follow the crowd depends on one basic economic variable, namely actual returns. Above average returns are reflected in a generally more optimistic attitude that fosters the disposition to overtake others' bullish beliefs and *vice versa*. This economic influence makes bubbles transient phenomena and leads to repeated fluctuations around fundamental values.

For a long time, the thinking about the funtioning of stock markets in the economics profession was dominated by the Efficient Market Hypothesis (EMH). Recent empirical investigations as well as factual developments have, however, eroded the trust in a theory which denies the existence of any systemic deviations of stock prices from their fundamental values. From the empirical side, one of the most discussed facts that gives rise to doubts in the overall efficiency of stock markets is the finding that stock prices exhibit more volatility than fundamentals or expected returns do. The volatility debate has recently been summarised and evaluated by West (1988). He finds evidence in favour of the excess volatility hypothesis to be persuasive and states that it cannot be explained adequately by standard models of expected returns or rational bubbles. Being not compatible with the random walk models suggested by the EMH, the finding of excess volatility points to intrinsic dynamic forces of speculative markets not related to fundamental factors. In his conclusion West, therefore, suggests that it might be necessary to consider 'non-standard' models focusing on fads and sociological or psychological mechanisms. A related empirical finding is the recent evidence on mean-reversion in asset prices (e.g. Poterba and Summers, 1988, and references therein). Technically, this means that there is positive autocorrelation over short horizons and negative autocorrelation over longer intervals in the data. A possible explanation is speculative overshooting of the price trend which is gradually eliminated beyond some range. Poterba and Summers, too, propose fads models to understand this regularity. The case for behavioural models of financial markets was already made emphatically by Shiller (1984), where also some hints are given concerning relevant material in other sciences. Another important author to be mentioned here is Kindleberger (1989). Throughout his penetrating book he highlights the importance of psychological factors and

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irrational factors in explaining historical financial crises.[1] Following these authors, the aim in this paper is to construct an elementary model of stock market dynamics which explicitly includes contagion of opinion and behaviour and to offer a behavioural explanation for the empirical findings discussed above.

Of course, the search for a theory which includes some kind of non-rational behaviour has not gone unrecognised by economic and financial theorists and a new paradigm began gradually to arise in the past few years under the heading of 'noise trader' models. This line of research is characterised by the introduction of traders which in some way deviate from a perfectly rational scheme of behaviour. These agents appear as naive traders, noise traders or chartists. Some authors (DeLong *et al.* 1990, 1991) assume that noise traders misperceive expected returns, others describe their behaviour as following a simple feedback rule and study the resulting dynamics of the market (Day and Huang, 1990; Genotte and Leland, 1990; Chiarella, 1992).

The way in which this paper aims at contributing to this body of literature is that the psychological factors which influence the behaviour of non-sophisticated traders will be modelled explicitly. This means that a formalism is introduced which describes the formation of expectations by those who are not fully informed about fundamentals. These expectations depend mainly (perhaps among other things) on the behaviour and expectations of others. Thus, what will be modelled is the process of mutual mimetic contagion among speculators. The key mechanism is similar to that described in Kirman's (1993) recent formalisation of recruitment in ant populations, which Kirman suggested to be also of relevance for the analysis of social dynamics in speculative markets. The mechanism introduced below seems to be also consistent with Topol's (1991) theory of mimetic contagion, where the agents try to trace out information about fundamentals from the bid and ask prices of others (who, however, may be as uninformed as they are themselves). The 'mechanics' of the contagion process are laid out in Section I. In Section II a more complete description of the market process is developed by adding a 'fundamentalist' group of traders and deriving the dynamics of prices. Conditions are given under which contagion may lead to the existence of (positive or negative) bubbles, i.e. stationary states where actual prices exceed fundamental values or are below them.

Bull and bear markets are, however, not stationary in reality. Bubbles grow up and burst, and periods where assets are undervalued or overvalued do not last forever. So it seems to be more appropriate to model them as transient phenomena. Section III, therefore, develops a model where switches between bull and bear markets occur periodically. The reason for this oscillation is that we allow additional *economic* factors to influence the process of opinion formation. To be precise, a slowly changing optimistic or pessimistic bias is

[1] The following quotation from the preface to the second edition seems to be characteristic for his position: '...the dismissal of conventional explanations of historical events with the remark that they violate the assumptions of economic analysis [i.e. full rationality] is infuriating...it is time that economics accept reality. (Kindleberger, 1989, p. xiii).

added to the contagion process. This variable for the overall disposition of the market depends on the only hard information available to naive speculators: the development of their *actual returns* including capital gains. Hence, speculators are not simply blind followers of the crowd: they quickly react on others' behaviour in order not to miss profit opportunities, but they also try to find out whether prevailing optimism or pessimism has a firm grounding in the market's actual development. The consequence is that once the pool of additional buyers in a bull market is exhausted and price increases diminish a gradual erosion of confidence in the validity of bullish beliefs occurs. This ends with a crash, and the game is repeated with reversed signs. Section IV concludes and points to possible extensions of the present approach.

I. AN ELEMENTARY FORMALISATION OF CONTAGION

The present section is mainly concerned with the determination of the behaviour of those traders who do not have access to information about fundamental values. In the absence of any piece of such information they necessarily have to rely on what can be observed on the market as the only base of their actions. Though I do not intend to discuss here what kinds of behaviour can be designated as 'rational', I should emphasise that following others' opinion is not irrational as long as there is no other source of information (see Orléan (1989) and Lesourne (1992) for intensive discussion).[2] If we accept this extreme assumption as an accurate description of the information set of a considerable part of traders, then a first conclusion could be that a speculator will be more willing to buy (sell) if he sees most traders buying (selling). The reason is that others' behaviour may presumably be influenced by better information about future developments of the market and may thus reveal information. As already mentioned such conjectures may be false, but nevertheless may lead to self-reinforcing fluctuations. The underlying process of contagion will be formalised by referring to the concept of synergetics originally developed in elementary particle and laser physics (see e.g. Haken, 1983) and applied to various problems from the social sciences by Weidlich and Haag (1983) among others.[3] 'Synergetics' basically consists of a probabilistic, macroscopic approach to the analysis of the dynamics of multi-component systems with interactions among the units constituting the system. Here the units of the system are the speculators operating in the market for some asset and their interactions are to be seen in the mutual infection with attitudes and opinions.

The basic set-up is as follows: as a simplifying assumption it will be postulated that a fixed number $2N$ of speculative traders exists. These may either be optimistic or pessimistic about the future development of the market.

[2] Scharfstein and Stein (1990) show that herd behaviour may be rational *despite better knowledge*. The reason is that 'following the herd' may be preferable under considerations about the reputation of managers.

[3] There have been some attempts to integrate such an analysis of non-rational opinion formation into business cycle models: the interested reader is referred to the 'Schumpeter clock' model by Haag *et al.* (1987), or to Kraft *et al.*'s (1986) reconsideration of Spiethoff's business cycle theory. Weidlich and Braun (1992) give an introduction to the ideas of synergetics directed to an economics audience.

More concretely, this means that they expect the price to increase or to fall, but this implementation of 'optimism' vs. 'pessimism' will play no role in the present section. Let us denote the number of optimistic or pessimistic investors, i.e. buyers or sellers, by n_+ and n_-, respectively. Of course, $n_+ + n_- = 2N$ (we do not allow for 'neutral' subjects). Defining: $n \equiv 0.5\ (n_+ - n_-)$ and $x \equiv n/N$ we have an index describing the average opinion of speculative investors, $x \in [-1, 1]$. It follows that $x = 0$ corresponds to a situation of balanced dispositions, i.e. there exists an equal number of optimistic or pessimistic individuals. Hence, situations with $x > (<)\ 0$ exhibit more or less predominant optimism (pessimism). In the extreme cases, $x = 1$ or $x = -1$ all agents have the same opinion (and to forestall some of the next section's content all would pursue the same strategy, i.e. all speculative traders would try to buy or all would try to sell). As long as situations $x \neq 0$ are not correlated to fundamentals they obey the usual definition of bubbles.

For the moment we will concentrate on the contagion process among speculators. Infection of attitudes will now be made explicit: with a high portion of optimistic traders, it would be very probable that the few remaining pessimistic ones would also change their attitude and buy stocks. The same is to be expected with reversed signs. Hence one may postulate probabilities exist for a pessimistic trader to become optimistic, say p_{+-}, and *vice versa*, p_{-+}. With contagion both probabilities should depend on the actual distribution of attitudes captured by the index x or the number n:

$$p_{-+} = p_{-+}(x) = p_{-+}(n/N), \quad p_{+-} = p_{+-}(x) = p_{+-}(n/N). \tag{1}$$

Note that this formulation implies all other individuals influence one particular speculator in the same way. This excludes the existence of financial gurus whose statements attract exceptional attention by others. Another simplifying assumption is that every individual may change his opinion only once at any one time. Assuming furthermore that the transition probabilities are the same for all actors and considering a large population of speculators the number of actual transitions from one subgroup to the other (i.e. between n_+ and n_-) can be approximated by the product 'members of subgroup times probability to change to another subgroup'. Thus, the change in the composition of the population of naive speculators can be determined in the following way: those who are pessimistic turn to an optimistic attitude with probability p_{+-}. Consequently, we expect a fraction $n_- p_{+-}$ to switch from the n_- to the n_+ group. *Vice versa*, with the probability p_{-+} a bullish speculator is infected with a negative disposition implying that approximately a fraction $n_+ p_{-+}$ of this population will change their trading strategy. From this it follows that the change in time of the number of optimistic and pessimistic traders is: $dn_+/dt = n_- p_{+-} - n_+ p_{-+}$ and $dn_-/dt = n_+ p_{-+} - n_- p_{+-}$. From the definitions of n and x we obtain:

$$\begin{aligned} dx/dt &= [(N-n)\, p_{+-}(x) - (N+n)\, p_{-+}(x)]/N \\ &= (1-x)\, p_{+-}(x) - (1+x)\, p_{-+}(x). \end{aligned} \tag{2}$$

In the Appendix I show how (2) can be derived more formally as an approximative mean value equation for the original stochastic system using the Master equation approach. Confining the analysis to mean values neglects the intrinsic dynamics of variances and higher moments, but is sufficient in order to determine the most probable development from any initial state. Technically, it can be justified by the convenient assumption of a sharply peaked *initial* distribution (see the Appendix for more details).

To obtain more insights into the dynamics that (2) potentially describes the transition probabilities will be specified. Note first what requirements p_{+-} and p_{-+} have to meet: first, all transition probabilities have to be positive by definition; second, to grasp the very idea of contagion the probability for a transition from pessimistic to optimistic attitude is larger than in the opposite direction if the prevailing disposition of the population is already optimistic and *vice versa.* Moreover, it seems reasonable to assume that $dp_{+-}/p_{+-} = a\,dx$, i.e. the relative change in the probability to switch from pessimism to optimism increases linearly with changes in x. Symmetric behaviour in both directions leads to $dp_{-+}/p_{-+} = -a\,dx$. These assumptions may suggest the following functional form commonly chosen in the related literature:

$$p_{+-}(x) = v\,e^{ax}, \quad p_{-+}(x) = v\,e^{-ax}. \tag{3}$$

Here, a gives a measure for the strength of infection or herd behaviour, v is a variable for the speed of change. One may note that this formulation allows for switches even in the absence of external forcing, i.e. with a balanced disposition ($x = 0$) we have $p_{+-} = p_{-+} = v > 0$. This means changes of attitudes occur due to personal circumstances not comprised in the model leading to minor fluctuations of the composition of the optimistic/pessimistic group even in stationary states.

With this specification of transition rates the time development of the mean value of the index x becomes:

$$\begin{aligned} dx/dt &= (1-x)\,v\,e^{ax} - (1+x)\,v\,e^{-ax} = 2v[\text{Sinh}\,(ax) - x\,\text{Cosh}\,(ax)] \\ &= 2v[\text{Tanh}\,(ax) - x]\,\text{Cosh}\,(ax). \end{aligned} \tag{4}$$

The transformations follow by the definition of the hyperbolic sine and cosine. This is now a description of a pure contagion dynamics. The outcome of this differential equation is well-known from other applications of the same ideas and is summarised in Proposition 1.

PROPOSITION 1: (i) *For* $a \leqslant 1$, (4) *possesses a unique stable equilibrium at* $x = 0$. (ii) *For* $a > 1$, *the equilibrium* $x = 0$ *is unstable and two additional, stable equilibria, say* $x_+ > 0$, $x_- < 0$ *exist* ($x_+ = -x_-$).

The results are easily proved by considering the equilibrium condition Tanh $(ax) = x$. Proposition 1 says that if the herd effect is relatively weak, then all deflections into one direction will die out in the course of events and the system will return to a state of balanced dispositions after some disturbance (see Fig. 1 with $a = 0{\cdot}8$ as an illustration). For $a > 1$, on the other hand, small deviations from the balanced state are sufficient to make a majority of traders bullish or bearish through mutual infection. Hence, the equilibrium $x = 0$ becomes

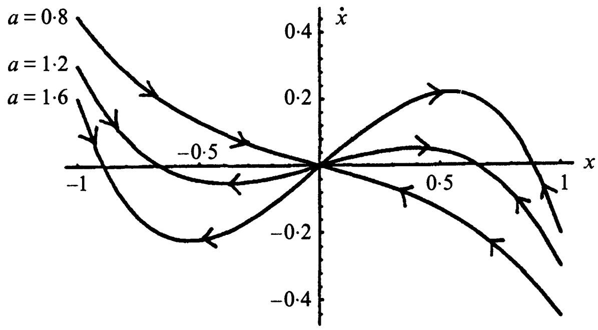

Fig. 1. Pure contagion dynamics.

unstable. Any small deflection from it will result in a snowball-like cumulative infection process. This dynamics leads to stationary states (x_+ or x_-) where the majority shares either an optimistic or a pessimistic opinion. Moreover, the higher the readiness to react on others (i.e. the higher a is), the larger will $x_\pm$ be in absolute value.

It can be shown that the bifurcation of the deterministic system corresponds to a bifurcation from a unimodal stationary probability distribution to a bimodal one in the stochastic system (Weidlich and Haag, 1983, ch. 2). Hence, in the case $a > 1$, the probability mass will be centred on x_+ and x_-, but there are positive probabilities of transition from x_+ to x_- and *vice versa*. Accordingly, there will occur at times switches between positive and negative equilibria after extreme realisations of the stochastic process. The 'mean escape time' from one equilibrium depends, however, inversely on the number of traders. As a consequence, bubble equilibria as described above will be very persistent in markets with a large number of participants. This suggests that merely stochastic fluctuations may be an improbable explanation for the volatility observed in reality. As a consequence, we will seek for endogenous mechanisms generating reversals of the disposition of traders.

II. CONTAGION AND PRICE DYNAMICS

It has already been mentioned that the mechanism of contagion alone does not provide a satisfying description of stock market dynamics. In order to set up a model of the market dynamics we first consider the demand and supply resulting from the disposition of speculators. It seems natural to assume that optimistic individuals will buy additional units of the asset in question because they expect a 'rising' market. On the other hand, pessimists fearing a downrush of prices will reduce the portion of the asset in their portfolio and will, consequently, enter on the supply side. To make the following analysis operable it will be assumed, in any instant, every speculative trader may either buy or sell a fixed amount of stock (t_N). This assumption allows us to express net excess demand of speculators (D_N) as:

$$D_N = n_+ t_N - n_- t_N = 2nt_N. \tag{5}$$

From the definition of x it follows that $n = Nx$ and (5) can be reduced to:

$$D_N = 2Nxt_N = xT_N, \quad T_N \equiv 2Nt_N, \tag{6}$$

where T_N denotes the trading volume of speculative investors. Only in the case $x = 0$ can all trades of speculators be carried out within this group. If this is not the case, we need at least one second group of actors that would possibly be willing to buy from and sell to speculators in order to close the model. Following the literature on 'noise traders' surveyed in the introduction we will introduce a second group denoted fundamentalists.

Excess demand of this second group depends on the difference between fundamental value (p_f) and actual price (p). A most elementary linear specification is adopted here:[4]

$$D_F = T_F(p_f - p), \quad T_F > 0. \tag{7}$$

T_F is a measure for the trading volume of fundamentalists as opposed to T_N. Concerning price dynamics one may either follow a market-clearing device or assume that prices adjust in finite time in the presence of excess demand or supply. In the latter case we may utilise the construction of a market maker to avoid temporary rationing. As in Day and Huang the 'market-maker' is suposed to match demand and supply in any instant and alternate prices in the usual direction. This leads to the following dynamic law:

$$dp/dt = \beta(D_N + D_F) = \beta[xT_N + T_F(p_f - p)]. \tag{8}$$

Here β is to be interpreted as a speed of adjustment coefficient. Market clearing would lead to the following equilibrium price depending on the average disposition of speculators: $p^* = (T_N/T_F)\,x + p_f$. Combining the contagion and price dynamics it seems reasonable to include a feedback effect from the price changes on the disposition of speculators. Thus information is not only drawn from the behaviour of others but also from the observed current price dynamics. A more general formulation of transition probabilities is then (the dot denotes the time derivative):

$$\mathrm{p}_{+-} = v \exp(a_1 \dot{p}/v + a_2 x), \quad \mathrm{p}_{-+} = v \exp(-a_1 \dot{p}/v - a_2 x). \tag{9}$$

The former parameter a has now been split into two parts: a_1 is a weight factor describing how much information investors try to draw from prices as opposed to that drawn from the behaviour of others (weighted by a_2). Dividing by v in the expression giving the influence of price changes is required for the following reason: the dynamics of the contagion process and the dynamics of prices have

[4] One may rationalise this in the following way: one-period returns are in general given as: Returns = $\delta[\mathrm{E}(\dot{p}) + r]/p$; E is the expectation operator, δ the discount factor, r nominal dividends and $\dot{p}$ price changes. Now assume agents entertain a (fundamentalist) model of price formation: $\dot{p}/p = t_f(p_f - p)$, i.e. they expect the price to revert to its fundamental after any deviation. This leads to: Returns = $\delta[t_f(p_f - p) + r/p]$. The price formation model above also implies that these agents consider the average price to equal p_f over longer intervals. Since dividends are paid out infrequently, they may evaluate real dividends using this average instead of the short-run price p. If, furthermore, real equilibrium dividends r/p_f equal the risk-adjusted average return in the economy, R, excess returns to be gained by arbitrage are given by $\delta t_f(p_f - p)$. Setting $T_F = \delta t_f$ this gives the expression in the main text. Hence, whether the extent of arbitrage by fundamentalists is sufficient to eliminate deviations or not (cf. Propositions 2 and 3) depends among other variables on their discount rate.

different (mean) time lags, $1/v$ and $1/\beta$, respectively. In order to capture accurately the influence of price changes on opinion formation, we have to consider the price change *taking place during* the time that is needed for the configuration itself to change. The entire dynamic system covering contagion and price formation reads:

$$\begin{aligned} \dot{x} &= 2v[\text{Tanh}\,(a_1\dot{p}/v + a_2 x) - x]\,\text{Cosh}\,(a_1\dot{p}/v + a_2 x) \\ \dot{p} &= \beta[xT_N + T_F(p_f - p)]. \end{aligned} \tag{10}$$

Of course, the dependence of investment behaviour on price dynamics (which is at the heart of most of the contributions in the noise-trader literature) enforces the contagion effect. System (10) has an astonishingly rich variety of dynamic behaviour. Analytically accessible results are collected in Proposition 2.

PROPOSITION 2: (i) *For* $a_2 \leqslant 1$ *there exists a unique equilibrium* $E_0 = (0, p_f)$. *For* $a_2 > 1$ *two additional equilibria,* $E_+ = (x_+, p_+)$ *and* $E_- = (x_-, p_-)$, *emerge (again:* $x_- = -x_+$ *and also* $p_f - p_- = p_+ - p_f$). (ii) *If* $E_\pm$ *exist,* E_0 *is always unstable.* (iii) *If* E_0 *is a unique equilibrium (i.e.* $a_2 < 1$), *it may either be stable or unstable. The condition for stability is given by:* $2[a_1\beta T_N + v(a_2 - 1)] - \beta T_F < 0$. (iv) *If* E_0 *is unique and unstable, at least one stable limit cycle exists and all trajectories of the system converge to a periodic orbit.*

An unpublished Appendix containing the proofs of this Proposition and the following one is available upon request. Fig. 2 gives an illustration of the stable

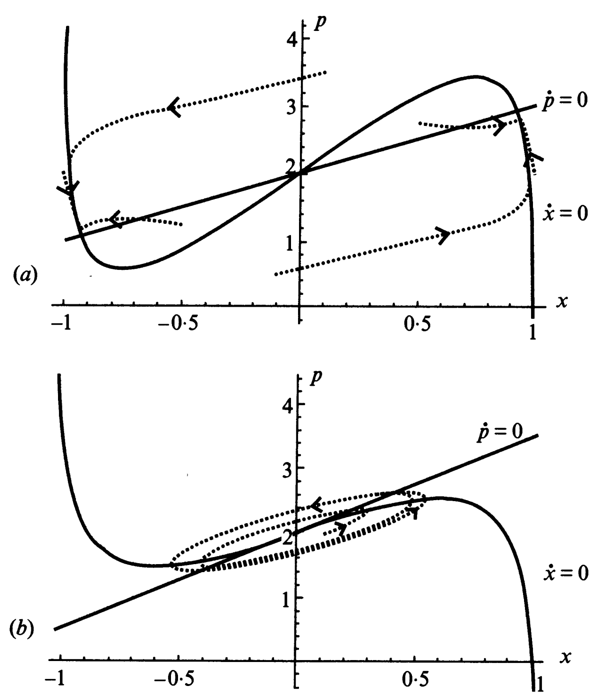

Fig. 2. Contagion and price dynamics. (*a*), Stationary bubbles; (*b*), cycles.

bubble and cyclical outcomes. It should be noted even though the condition for existence of bubble equilibria is the same as in the pure contagion case, the stability condition for the $x = 0$ equilibrium is rather stronger now, implying potential instability even in some cases where $a_2 < 1$. In particular, one infers from Proposition 2 that high values of T_N, v, and a_1 and low values of T_F favour instability. The levels of x_+ and x_- are the same as in the pure contagion case of the preceding section. So one can say that the price dynamics influence only the transient part of the motion. At the stationary states, the market clearing price is above or below the fundamental value. Note that this implies transactions occur between chartists and fundamentalists. Either, for $x_+ > 0$ chartists buy additional units of the asset which are sold by fundamentalists or, if $x_- < 0$ prevails, the noise traders fear a declining market and sale to fundamentalists at a price below the fundamental value.

According to item (iv) a cyclic motion prevails if both the local stability condition of the fundamental equilibrium is violated and a_2 does not exceed the benchmark value 1. Then an endogenous switching between overvaluation and undervaluation occurs. Though the reaction on others' behaviour (measured by a_2) is not strong enough in this case to create stationary majorities with optimistic or pessimistic disposition, the resulting price reaction after some disturbance destabilises the fundamental equilibrium: thus, during the upswing of the cycle, traders react both on positive changes of x as well as on the accompanying price increase. However, after some time, the infection process will have reached its climax and will diminish thereafter. This, on the other hand, leads to a lower excess demand by chartists generating a reversal of the price trend, too (see the counter-clockwise motion in Fig. 2*b*). In the downward part, direct contagion of fear and the aggravating influence of a negative price trend reinforce each other. Again this joint causation is too strong to allow convergence to the unique fundamental equilibrium.

The information in Proposition 2 does not completely characterise the global behaviour of system (10). For example, I have not given explicit stability conditions for bubble equilibria because working out their implications requires some additional technical efforts. Numerical experiments suggest that the bubble equilibria will be stable for a broad range of parameter values but not for all. Simulation runs also indicate, that stable equilibria $x_\pm$ can coexist together with a limit cycle enclosing all three steady states which is also stable. Taking the randomness that has been suppressed in our derivations explicitly into account, the system may then undergo transitions between cyclic behaviour and stable steady states leading to an erratic appearance of the overall evolution of prices.

III. SWITCHING BETWEEN BEAR AND BULL MARKETS

In the preceding sections the concern was how bubbles get started and how they evolve. Though a cyclic scenario was identified for a limited range of parameter values, strong reactions on others' behaviour above the benchmark value $a_2 = 1$ give rise to persistent bubbles. In this part of the paper it will be shown how the analytical set-up can be expanded to encompass also an endogenous mechanism responsible for the ultimate breakdown of such bubbles. Or, expressed in a more technical fashion we try to explain 'mean-reversion' in what follows.

In Kindleberger's theory the period of distress precedes the ultimate crash. For this period '...a change in expectations from a state of confidence to one lacking confidence in the future is central' (Kindleberger, 1989, p. 109). This corresponds to numerous statements in informal discussions that an ultimate crash is foreshadowed by a gradual erosion of confidence in the market. As an example, Shiller (1988) presents evidence of pessimistic and warning voices on the eve of the October 1987 crash.

This section attempts to model this change in expectations leading to a period of 'distress', which in turn is eventually ended by a stock market crash. In Sections I and II we showed that contagion may lead to a predominant optimistic or pessimistic sentiment among the group of speculative investors generating steady states where stock prices are either above or below fundamental values. Though there have often been prolonged periods of time where financial experts have judged that stocks are overvalued or priced too low, such phases have ultimately found an end, often through a catastrophic crash (as in October 1987). What we are looking for here is an endogenous mechanism leading to a reversal of the speculators' opinion. I propose to formulate the above propositions in the following way: a variable is formally added which captures a general prevailing mood of the market. If the variable is positive then it may enforce the contagion mechanism leading to a higher transition probability than otherwise. If the variable is negative, then it similarly weakens the impact of contagion (people are less willing to overtake others' optimism if they have some reason to believe in adverse developments). This variable will be denoted a_0. Introducing it into the determination of transition probabilities gives:

$$\begin{aligned} \mathrm{p}_{+-} &= v \exp\left(a_0 + a_1 \dot{p}/v + a_2 x\right), \\ \mathrm{p}_{-+} &= v \exp\left(-a_0 - a_1 \dot{p}/v - a_2 x\right). \end{aligned} \tag{11}$$

Leaving aside the price component for the moment, one may look back at Fig. 1 to get an idea of how this additional factor changes the outcome of the contagion process. Positive (negative) values of a_0 will shift all the curves upwards (downwards). As an immediate consequence, even with a moderate infection rate a_2, there will be a unique equilibrium with a preponderance of buyers (sellers) and an equilibrium price exceeding (or lying below) the fundamental value. In the event of multiple equilibria ($a_2 > 1$), both x_+ and x_- are moved to the right (left). As is intuitively plausible, for high (absolute) values of a_0, the lower (or upper) and middle equilibria disappear and one ends up with a unique equilibrium x_+ close to 1 (x_- close to -1). In summary, the fact that positive 'basic dispositions' foster bullish reactions and weaken contagion of fear is mirrored in larger optimistic majorities and smaller pessimistic majorities than otherwise, whereas prevailing negative sentiments have the opposite effect. How can such a factor a_0 be determined? The assumption here is that a_0 is related to a basic economic variable, namely actual returns (including capital gains) compared to average expected returns. Actual returns are given by $(r+\dot{p})/p$, where r is the constant nominal dividend payment. Denote the expected (real) rate of return R. We exclude noise in the rate of return since it would not alter any of our main conclusions. Then a_0 may increase (decrease) if $(r+\dot{p})/p$ is above (below) R. In this way contagion is not fully decoupled from basic economic variables, as is essentially the case in the preceding sections. Formulated somewhat differently one can also say that by looking at the development of returns, agents try to get some feeling about whether the market got out of its depth. Since this 'basic mood of the market' may change gradually we can formulate the following dynamic law:

$$da_0/dt = \tau[(r+\tau^{-1}\dot{p})/p - R]. \tag{12}$$

Again, τ is to be interpreted as an adjustment coefficient. By the same logic as was laid out in the preceding section, the capital gain has to be set in relation to the mean time lag in the motion of a_0; thus one has to divide by τ. In order to be able to handle this dynamic in two dimensions only, we suppose instantaneous market clearing. This gives $p = p_f + (T_N/T_F)\,x$ and $\dot{p} = (T_N/T_F)\,\dot{x}$. Dropping the effect of changing prices on transition probabilities for simplicity,[5] the joint dynamics of x and a_0 reads:

$$\begin{aligned} \dot{x} &= 2v[\operatorname{Tanh}(a_0 + a_2 x) - x]\operatorname{Cosh}(a_0 + a_2 x) \\ \dot{a}_0 &= \tau\{[r+\tau^{-1}(T_N/T_F)\,\dot{x}]/[p_f + (T_N/T_F)\,x] - R\}. \end{aligned} \tag{13}$$

Furthermore, it is assumed $r/p_f = R$, i.e. there is no systematic deviation in profit expectations from their equilibrium values (allowing different values for R would only displace the equilibrium without changing any of the basic results). Proposition 3 gives the main results on the dynamics of system (13):

PROPOSITION 3: (i) *The dynamics* (13) *always possesses the unique equilibrium* $E = (0, 0)$. (ii) *The equilibrium is stable (unstable), iff* $a_2 - 1 + (T_N/T_F)/p_f < (>)\ 0$. (iii) *If the equilibrium is unstable, at least one stable limit cycle exists and all trajectories of the system converge to a periodic orbit.*

Again, the unique equilibrium corresponds to a situation where the price equals the fundamental (from $x = 0$ it follows that $p = p_f$) and there is a balanced atmosphere in the market ($a_0 = 0$). Note that the stability condition is again stronger than in the one-dimensional dynamics. Figs. 3*a*, *b* give graphical representations of the cyclical dynamics. First, Fig. 3*a* may provide some straightforward intuition about what is happening. Here graphs of $\dot{x} = 2v[\operatorname{Tanh}(a_0 + a_2 x) - x]\operatorname{Cosh}(a_0 + 2_2 x)$ for different values of a_0 and $a_2 > 0$ are given. Beginning with a situation without any bias in the disposition of investors ($a_0 = 0$), we may assume that due to some random event a positive bubble has occurred and the market is described by the right-hand intersection with the abscissa. With the resulting high stock prices real rates of return become lower than expected on average. The ensuing erosion of optimism leads

[5] The results of a three-dimensional dynamics including sluggish price adjustment and feedback from price changes on individual behaviour are not very different from those of system (13); a short account is given in the concluding section.

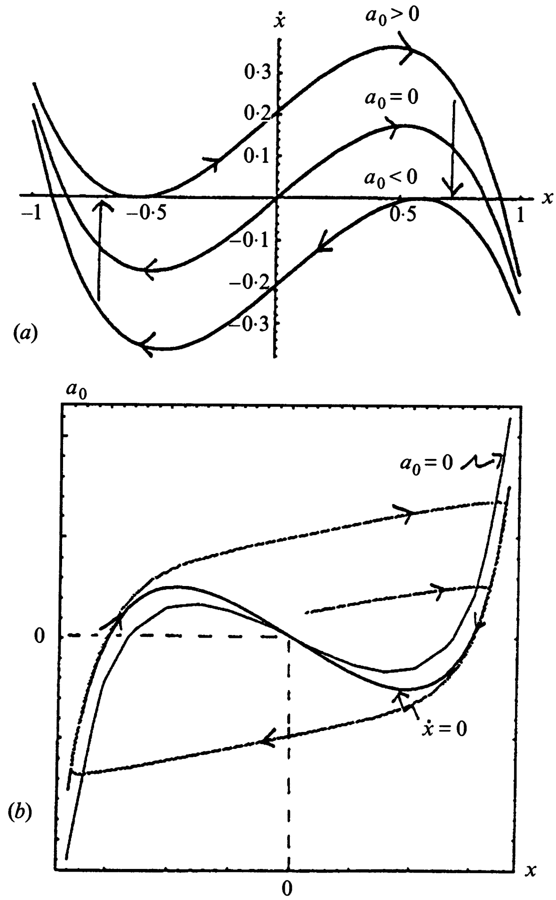

Fig. 3. (*a*). Shifts of contagion dynamics with positive/negative bias; (*b*), phase diagram.

to a downward shift of the curve and the portion of optimistic traders is gradually reduced. The process continues in this way until the equilibrium becomes tangential to the abscissa (it collapses with the middle one). Any further reduction of a_0 leads to a shift to the unique left-hand equilibrium. Literally, the slow 'basic sentiment' change caused by insufficient returns has reached an extent where the warning and pessimistic voices have become so loud that they infect the mass of unsophisticated traders. This leads to a crash that ends in the negative bubble and now stocks are undervalued. The whole story repeats the other way round once traders recognise that they earn unexpectedly high returns and a gradual recovery begins, which after some point again sets a positive bubble in motion.

This reasoning (which might formally be reminiscent of Kaldor's famous business cycle analysis) is replicated in terms of a phase diagram picture in Fig. 3*b*.[6] In the phase diagram one may observe that with rising stock prices a_0 also

[6] As shown in a technical Appendix (available upon request) both isoclines exhibit two extrema in the case $a_2 > 1$. Furthermore, $\dot{a}_0 = 0$ lies below (above) $\dot{x} = 0$ for negative (positive) values of x.

goes up for an extended period. The reason is that capital gains lead to increasing actual returns. Once infection has reached the overwhelming majority of speculative traders, a change in basic sentiment occurs because *the exhaustion of the pool of potential buyers causes price increases to diminish.* Due to the slow-down of the bubble a change in expectations occurs 'from a state of confidence to one lacking confidence'. After this turning point has been reached, the market may be said to be in the period of distress as described by Kindleberger. In the diagram, the period of distress is located on the upper right hand between the $\dot{a}_0 = 0$ and the $\dot{x} = 0$ isoclines. There the number of additionally infected speculators still rises for some short time, while there is already some scepticism spreading out (declining a_0) because of the deceleration of the price trend. However, the realisation of declining profit opportunities leads very soon to a complete collapse of the bubble and a cumulative downward motion in average opinion. Since this leads to increasing sales, prices go down as well reinforcing contagion of fear among traders. This continues until the majority is infected with pessimism. In the following the price decrease weakens and a recovery of returns leads to a similar reversion of the 'basic disposition' as described above. [7]

IV. CONCLUDING REMARKS

A model was set forth where contagion of opinions and behaviour on the stock market is made explicit. In summary, the paper has formulated a basic cyclical mechanism around fundamental values. Overvaluation (or undervaluation) of assets occurs because of fierce self-amplifying reactions of speculators on small deviations from the equilibrium. On the other hand, an endogenous breakdown of bubbles is brought about because excess profits vanish as the bubble decelerates. Hence, both excess volatility and mean-reversion can be explained with this type of noise trader/infection model.

We remind the reader that there exist several (microeconomic) explanations for the herd behaviour of speculative traders: first, they can be seen indeed as acting irrationally; second, contagion can be interpreted as an attempt to draw information from what the others do; and third, reputation considerations may urge even smart investors to follow the crowd. In reality, a wide variety of actors will be found. In order to take account of the observed heterogeneity of agents and complex micro-structure of speculative markets a probabilistic approach was adapted that proved to be useful already in other applications in the social sciences. The stochastic process in the main text (resp. its quasi-deterministic analogue) can then be understood as the macroscopic outcome of a multitude of micro-motives. In this perspective, the present model should *not* be seen as antagonistic to recent models which explain the rationality of herd

[7] Different degrees of attraction of additional traders play an important role in explaining historical bubbles and crashes. To grasp this fact, a more complex model could cover attraction of additional individuals in bullish periods (thus endogenising N). Presumably, this would *ceteris paribus* prolong the ongoing bubble; however, since there exists in any case only a finite pool of potential entrants, this would not prevent the mechanisms described above from being effective.

behaviour in certain environments, but rather as an attempt to work out typical overall patterns of market dynamics implied by contagion of opinions and behaviour.

There are some interesting avenues for exploring this kind of model further. In particular, embedding the basic mechanisms described in this paper in a richer dynamic structure may help to explain the finding of complex nonlinear dependence in financial market data.[8] Introducing sluggish price adjustment again into the framework of Section III provides a first step in this direction. Though the intuition of the process is not altered, the resulting three-dimensional system can exhibit a more complex time evolution: as has been shown by simulations (details are available upon request) the inherent periodicity of the process appears often in a more complicated fashion than possible in a two-dimensional model, i.e. a multiple of main periods (2, 3, ...) may exist. In a closely related model, where I allow for fluctuations among the chartist and fundamentalist groups according to the performance of their strategies, chaotic attractors could be identified (see Lux (1994) for details). It turned out, that the time paths sampled from the attractors look very well-behaved and allow us to explain one of the main stylised facts of speculative markets, namely the fat tails (leptokurtosis) of the return distributions. As another extension, it was suggested by one of the referees that the 'synergetic' modelling concept adopted here may also provide a framework for analysing the dynamics of the second moment (its time development can be approximated using the Master equation detailed in the Appendix). Since this may lead to insights into the generating mechanisms for the well-known stochastic ARCH-type nonlinearities in the data, it is an important subject on the agenda for future research.

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Appendix

In this technical appendix the Master equation formalism for the basic stochastic process of Section I is laid out. It will be shown that the deterministic differential equation (2) in the main text can be neatly interpreted as an approximation to the change in time of the mean value of the opinion index x in the original stochastic system.

First, we can deliberately choose the time scale such that there are only 'nearest neighbour transitions', i.e. that the probability of joint movements of a group of population members in any instant is negligible. Define the probability for the distribution of the population of speculators to change from e.g. some configuration $\{n_+, n_-\}$ to $\{n_+ - 1, n_- + 1\}$, or, equivalently, to change from $\{n\}$ to $\{n-1\}$:

$$w_{-+}(n/N) = n_+ \, \mathrm{p}_{-+}(n/N). \tag{A 1}$$

[8] Unfortunately, it does not seem possible to uncover a precise type of attractor governing financial markets despite apparent non-linearity in the data. This failure of identification may very likely be attributable to switches between attractors because of parameter non-stationarity or even switches between coexisting attractors if the parameters are constant over time (cf. the coexistence of fixed points and cycles in Section II).

Vice versa, the probability to move from $\{n\}$ to $\{n+1\}$ is:

$$w_{+-}(n/N) = n_{-}\,\mathrm{p}_{+-}(n/N). \tag{A 2}$$

In addition, if the population size is large enough to allow a continuous-time approximation one can write down the following so-called Master equation for the change of the probability distribution $\mathrm{P}(n;t)$ over time:

$$d\mathrm{P}(n;t)/dt = [w_{+-}(n-1)\,\mathrm{P}(n-1;t) + w_{-+}(n+1)\,\mathrm{P}(n+1;t)] - [w_{+-}(n)\,\mathrm{P}(n;t) + w_{-+}(n)\,\mathrm{P}(n;t)]. \tag{A 3}$$

In (A 3) the first bracket on the RHS covers the flux from neighbouring states to some state $\{n\}$, the second bracket the probabilities to move in the opposite direction, i.e. to leave state $\{n\}$.

The mean value of the configuration $\{n\}$ is defined by:

$$\bar{n}_t = \sum_{n=-N}^{N} n\mathrm{P}(n;t). \tag{A 4}$$

Its change over time is given by:

$$\begin{aligned} d\bar{n}_t/dt &= \sum_{n=-N}^{N} nd\mathrm{P}(n;t)/dt \\ &= \sum_{n=-N}^{N} [w_{+-}(n) - w_{-+}(n)]\,\mathrm{P}(n;t) = \overline{w_{+-}(n) - w_{-+}(n)}. \end{aligned} \tag{A 5}$$

A similar equation can be derived for the variance of n.[9] Equation (A 5) requires knowledge of the full probability distribution in order to calculate the time derivative of $\bar{n}_t$. Assuming a sharply peaked maximum of the *initial* probability distribution $\mathrm{P}(n; t=0)$, one may approximate the RHS of (A 5) by the first term in the Taylor-series expansion around $\bar{n}_t$ yielding the closed expression:

$$d\bar{n}_t/dt = w_{+-}(\bar{n}) - w_{-+}(\bar{n}). \tag{A 6}$$

Dividing by N and inserting the definitions (A 1) and (A 2) one translates (A 6) into a dynamic equation for the mean value of the opinion index x which is:

$$\begin{aligned} d\bar{x}/dt &= [(N-\bar{n})\,\mathrm{p}_{+-}(\bar{n}/N) - (N+\bar{n})\,\mathrm{p}_{-+}(\bar{n}/N)]/N \\ &= (1-\bar{x})\,\mathrm{p}_{+-}(\bar{x}) - (1+\bar{x})\,\mathrm{p}_{-+}(\bar{x}). \end{aligned} \tag{A 7}$$

Suppressing the bars, (A 7) is the same as (2) in the main text. This operation has transformed the stochastic dynamics into a quasi-deterministic one which greatly simplifies analysis. Of course, the approximation is valid only as long as the assumption of a sharply peaked distribution is not violated. As is mentioned in the main text, this assumption cannot be maintained in the long run, if multiple equilibria or cycles exist (the probability distribution will then develop into a bi-modal or, in the case of periodicity in two dimensions, a stationary distribution with probability mass centred on the mean-value limit cycle). Nevertheless, the above approximation gives (1) the most probable short-run evolution from any initial condition and (2) for a large underlying population the general results on the dynamics will be analogous in the stochastic system.

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[9] See Weidlich and Haag (1983) for the concrete form and more details, e.g. the equivalence between the Master equation approach and the use of a Fokker–Planck or Langevin equation.

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